Junior High Number Sense – Special Topics

Competition in Spring 2023 and Spring 2024

This document contains information on some of the new tricks that will appear on the 2023 and 2024 Number Sense tests.

The new tricks are sectioned in the order that they will appear on the test. Some of the new tricks are given explicitly. For the others, the student is encouraged to search for an easy mental math formula, procedure for working the problem, or information on the topic.

Multiplication by 37 [#21-40]

In this problem, students will be asked to multiply 37 by a multiple of 3. Some examples include 18×37 and 51×37. The student is encouraged to work out an easy mental math trick for problems of this type. You might consider working several of these problems out and looking for the obvious pattern. Then, try to provide an algebraic proof that your conjecture is correct. Good luck!

Sum of Reciprocals [#41-60]

Problems of this type look like $\frac{1}{4} + \frac{1}{6}$, where you are adding the reciprocal of whole numbers. Algebraically, the format is $\frac{1}{a} + \frac{1}{b}$. Getting a common denominator gives

$$\frac{1}{a} \cdot \frac{b}{b} + \frac{1}{b} \cdot \frac{a}{a} = \frac{b}{ab} + \frac{a}{ab} = \frac{a+b}{ab}.$$

Thus, the sum of the reciprocals of integers is the sum of the integers divided by their product. However, what happens if this fraction can be reduced? Can you determine when a fraction can be reduced versus when the sum divided by the product will already be in lowest terms? The answer is yes! It all depends on the greatest common divisor (GCD) of the two integers a and b. If the GCD of a and b is 1, then $\frac{a+b}{ab}$ will be in lowest terms. If the GCD of a and b is a value d that is greater than 1, you reduce $\frac{a+b}{ab}$ to lowest terms by dividing both the numerator and denominator by d: $\frac{(a+b)/d}{ab/d}$.

Example:

$\frac{1}{8} + \frac{1}{12} =$

Solution:

Notice first that the GCD of 8 and 12 is 4. Take the sum of 8 and 12 and divide by 4: $(8 + 12)/4 = 5$. Next, take the product of 8 and 12 and divide by 4: $8(12)/4 = 24$. Thus, $\frac{1}{8} + \frac{1}{12} = \frac{5}{24}$.

Partial Fraction Decomposition [#61-80]

Partial Fraction Decomposition is when you take a single fraction and break it down into parts.

Example:
If $\dfrac{2x-1}{x^2-5x-14} = \dfrac{A}{x-7} + \dfrac{B}{x+2}$, what is the value of B?

Solution:
To see how to decomposition works, notice that the right side can be combined

$$\frac{A}{x-7} \cdot \frac{x+2}{x+2} + \frac{B}{x+2} \cdot \frac{x-7}{x-7} = \frac{A(x+2)+B(x-7)}{x^2-5x-14}.$$

Since this fraction is equal to $\dfrac{2x-1}{x^2-5x-14}$, the numerators of the two fractions must also be equal. This gives

$$A(x+2) + B(x-7) = 2x-1.$$

The trick is to pick a value of x that will get rid of the variable A and give an equation to solve for B. What value of x will wipe out the $A(x+2)$ term? The value $x=-2$ will work. How did we get $x=-2$? By solving for x when the factor that is multiplied by A is equal to 0: $x+2=0 \longrightarrow x=-2$.

Use this value of x in the equation above:

$$A(-2+2) + B(-2-7) = 2(-2)-1$$
$$-9B = -5$$
$$B = \frac{5}{9}$$

So, what's the trick? Pick the value x that gives a value of 0 in the denominator under the variable you are solving for. Evaluate this value in the numerator of the fraction and then denominator under the other variable. Divide these two numbers.

In this example, $x=-2$ is the value that gives 0 under the variable B. Evaluate $x=-2$ in the numerator and the other denominator: $B = \dfrac{2(-2)-1}{-2-7} = \dfrac{-5}{-9} = \dfrac{5}{9}$. With practice, this process can be done quickly.

Remainders in Different Bases [#61-80]

You probably know the trick for finding the remainder when dividing by 9: keep adding up the digits of the number until you get a single digit less than 9. For example, $416 \div 9$ has a remainder of 2 since $4+1+6=11$ and $1+1=2$. It turns out that this type of trick can be applied in other bases.

Example:
Find the remainder when 316_8 is divided by 7.

Solution:
The trick works when you are dividing by the last single digit in the base. The last single digit in base 10 is 9; the

last single digit in base 8 is 7. Continue adding up the digits in the number until you get a single digit less than the divisor. In this case, $3 + 1 + 6 = 12_8$ and this gives $1 + 2 = 3$. (Don't forget to do the sum in the base that is given!).

Notice also that you can "cast out" numbers. Since the 1 and 6 in 316 add up to 7, you can "cast" those out and are left with only the 3, which is the answer.

Example:

The remainder of $(23_5 \times 12_5 + 42_5) \div 4$ is __________.

Solution:

The same process is used on each individual number and then their results combined.

- $23_5 \div 4$ has a remainder of 1 (since $2 + 3 = 10_5$ and $1 + 0 = 1$)
- $12_5 \div 4$ has a remainder of 3 (since $1 + 2 = 3$)
- $42_5 \div 4$ has a remainder of 2 (since the "4" can be cast out).

Together, this reduces to $1 \times 3 + 2 = 10_5$ which has a remainder of 1 when divided by 4.

For practice questions, visit `www.academicmeet.com` and click on Resources.

This document was prepared by Doug Ray for competition, 2023 and 2024. If you have any questions about the material presented, please email `doug@academicmeet.com`.