# Junior High Number Sense – Special Topics

**Competition in 2026-2027 and 2027-2028**

This document contains information on some of the new tricks that will appear on the PSIA Number Sense tests for the 2026-2027 and 2027-2028 years.

The new tricks are sectioned in the order that they will appear on the test. Some of the new tricks are explained explicitly. For the others, the student is encouraged to search for an easy mental math formula, procedure for solving the problem, or information on the topic.

---

Metric Prefixes [#21-40]

You are probably familiar with most metric prefixes used near the base unit.

**Standard SI Prefixes**

| kilo (k) | hecto (h) | deka (da) | *base* | deci (d) | centi (c) | milli (m) |
|---|---|---|---|---|---|---|
| $10^3$ | $10^2$ | $10^1$ | $10^0$ | $10^{-1}$ | $10^{-2}$ | $10^{-3}$ |
| 1000 | 100 | 10 | 1 | 0.1 | 0.01 | 0.001 |

This Number Sense trick will use prefixes that are farther away from the base unit. They are used for really big numbers and really small numbers:

**Advanced SI Prefixes**

| tera (T) | giga (G) | mega (M) | $\cdots$ | micro ($\mu$) | nano (n) | pico (p) |
|---|---|---|---|---|---|---|
| $10^{12}$ | $10^9$ | $10^6$ | | $10^{-6}$ | $10^{-9}$ | $10^{-12}$ |

Notice that the exponent for these prefixes differ by 3, meaning the prefixes represent factors of 1000. In other words, using meters, 1 terameter = 1000 gigameters, and 1 gigameter = 1000 megameters, etc. The larger the exponent, the larger the unit. The abbreviations for the units are given in parentheses. They work the same as the units you are familiar with: nm = nanometers; GL = gigaliters. 1.5 Tm is 1.5 terameters. Notice that the prefix abbreviation for micro is different. This is the Greek letter $\mu$, mu.

Example:

8400 pm = _____ $\mu$m

Solution:

Here we are asked to convert from picometers ($10^{-12}$) to micrometers ($10^{-6}$). Since we are going from a smaller unit to a larger unit, we divide by 1,000,000. To divide by 1,000,000, move the decimal six places to the left. This gives .0084. [Remember not to use leading zeros in front of the decimal nor extra zeros on the right side!]

---

Example:
$2\frac{1}{2}$ gigaliters is equivalent to how many millions of liters?

Solution:
Since millions is $10^6$ and giga- means billions ($10^9$), we will multiply by 1000 to convert from gigaliters to megaliters. $2\frac{1}{2} \times 1000 = 2500$.

---

Remainder When Dividing By 11 [#41-60]

To find the remainder when dividing by 11, find the sum of every other digit starting in the one's place. Then, find the sum of the other digits. Subtract the first sum minus the second sum. This sum has the same remainder when divided by 11 as the original number.

Example:
381527 ÷ 11 has a remainder of _____

Solution:
Add every other number starting in the one's place: 7 + 5 + 8 = 20
Add all the other numbers: 2 + 1 + 3 = 6
Subtract: 20 – 6 = 14
You can probably see that 14 ÷ 11 will have a remainder of 3, but if not, you can repeat the process. Since there are only 2 digits, you just subtract them: 4 – 1 = 3.

Notes: The first sum might be smaller than the second sum. If so, just add or subtract 11s to make the numbers work. In fact, you can just "ignore" 11s as you go. For example, when finding the remainder when 91645 is divided by 11, the first numbers you add together are 5 + 6 which is 11. You can subtract that 11 off right away and keep going. Another method used by some Number Sensers is the alternating method: Start with the one's place digit, then subtract the ten's place digit, then add the hundred's place digit, etc. Continue adding then subtracting until you reach the end of the number.

---

Multiplication by 133 [#61-80]

This one is your homework assignment for this year! Hint: Look for a multiple of 133 that is close to a number that is *much* more manageable to work with. Then, develop the trick based on that number and how far the multiple of 133 is away from that number. Good luck!

---

### Arithmetic Mean-Geometric Mean Inequality [#61-80]

The *arithmetic mean* of two numbers is their sum divided by 2. It's the number we normally are referring to when we ask about the average. The *geometric mean* of two numbers is the square root of the product of two numbers.

The Arithmetic Mean-Geometric Mean Inequality (AM-GM inequality) states that the geometric mean of a set of positive numbers is no larger than the arithmetic mean of the same set of numbers. As an inequality with $x$ and $y$ representing two positive numbers, it looks like this:

**Arithmetic Mean-Geometric Mean Inequality**

$$\sqrt{xy} \leq \frac{x+y}{2}$$

This inequality can be extended to more numbers, but for the Number Sense test, we will only use two numbers at a time.

Example:

If $x, y \geq 0$ and $x + y = 10$, what is the largest possible value of $xy$? _____

Solution:

Since $\sqrt{xy} \leq \frac{x+y}{2}$ by the AM-GM inequality and we know that $x + y = 10$, we have $\sqrt{xy} \leq \frac{10}{2} = 5$. To relate this back to $xy$, just square both sides: $xy \leq 5^2 = 25$. The answer is 25.

---

Example:

If $x, y \geq 0$ and $3x + y = 16$, what is the largest possible value of $\sqrt{243xy}$? _____

Solution:

This one is more tricky! We need to get the inside of the square root to look like the two parts of the sum, $3x$ and $y$. We notice that $243 = 81 \times 3$ and 81 is a perfect square. We can take the 81 out of the square root as a 9 and then use the AM-GM inequality: $\sqrt{243xy} = \sqrt{81 \cdot 3xy} = 9\sqrt{3xy} = 9\sqrt{(3x)y} \leq 9 \cdot \frac{3x+y}{2} = 9 \cdot \frac{16}{2} = 9 \cdot 8 = 72$

---

Practice Questions – The following practice questions cover the above examples and should be used to guide your inquiries into the new types of questions to be asked on the Number Sense tests.

1. 43000 gigameters = _____ terameters
2. $6\frac{1}{4}$ $\mu$L = _____ pL
3. 3728 nm = _____ mm
4. How many meganewtons are equivalent to two million kilonewtons? _____
5. 93728 ÷ 11 has a remainder of _____
6. 248019 ÷ 11 has a remainder of _____
7. 208759 ÷ 11 has a remainder of _____
8. 419272 ÷ 11 has a remainder of _____
9. $133 \times 12$ = _____
10. $133 \times 32$ = _____
11. $84 \times 133$ = _____
12. $56 \times 133$ = _____
13. If $x, y \geq 0$ and $x + y = 20$, what is the largest possible value of $xy$? _____
14. If $x, y \geq 0$ and $x + y = 38$, what is the largest possible value of $xy$? _____
15. If $x, y \geq 0$ and $2x + y = 12$, what is the largest possible value of $xy$? _____
16. If $x, y \geq 0$ and $12x + 3y = 24$, what is the largest possible value of $6xy$? _____

Answers: **1.** 43 **2.** 6,250,000 **3.** .003728 **4.** 2000 **5.** 8 **6.** 2 **7.** 1 **8.** 7
**9.** 1596 **10.** 4256 **11.** 11172 **12.** 7448 **13.** 100 **14.** 361 **15.** 18 **16.** 24