Junior High Number Sense – Special Topics

Competition in Spring 2025 and Spring 2026

This document contains information on some of the new tricks that will appear on the 2025 and 2026 Number Sense tests.

The new tricks are sectioned in the order that they will appear on the test. Some of the new tricks are given explicitly. For the others, the student is encouraged to search for an easy mental math formula, procedure for working the problem, or information on the topic.

Multiplication by Numbers That Straddle 100 [#21-40]

In this problem, students will be asked to multiply two numbers near 100, one less than 100 and one greater than 100. For example, 97×102, since one number is smaller than 100 and one number is larger than 100.

Here is the algebraic derivation of this trick:

Let a represent how much smaller one number is from 100 and let b represent how much larger one number is from 100.	$(100 - a) \times (100 + b)$
Multiply out these terms.	$10000 + 100b - 100a - ab$
Simplify.	$10000 + (b - a)(100) - ab$
Add and subtract 100 (to keep the expression the same).	$10000 + (b - a)(100) - ab + 100 - 100$
Group the "minus 100" with the middle term and the "plus 100" in with the ab term.	$10000 + (b - a - 1)(100) + (100 - ab)$
Factor 10,000 as 100(100).	$100(100) + (b - a - 1)(100) + (100 - ab)$
Then, factor out 100.	$[100 + (b - a - 1)](100) + (100 - ab)$
Identity place values.	$\underbrace{[100 + (b - a - 1)]}_{\text{Hundred's Place Value}}(100) + \underbrace{(100 - ab)}_{\text{One's Place Value}}$ (1)
The last two digits are 100 minus the product of the differences from the original numbers.	$100 - ab$
To compute the first two or three digits, find $b - a - 1$ and add to 100.	$100 + (b - a - 1)$
Notice that these first digits are equivalent to taking the number greater than 100 and subtracting the difference from 100 of the smaller number and subtracting an additional 1.	$(100 + b) - a - 1$

First Digits	Last Digits
$(100 + b) - a - 1$	$100 - ab$

Example:

$97 \times 102 =$

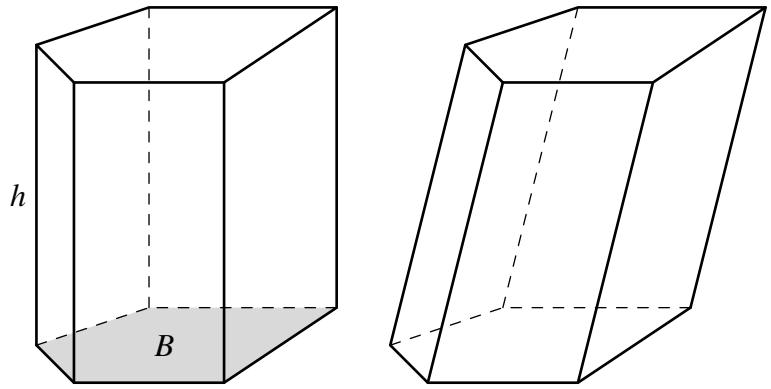
Solution:

Since 97 is 3 less than 100, $a = 3$ and since 102 is 2 more than 100, $b = 2$. Compute $100 - ab = 100 - (3)(2) = 100 - 6 = 94$. The last two digits are 94.

Compute $b - a - 1$: $b - a - 1 = 2 - 3 - 1 = -2$. Subtract 2 from 100 to get 98 and these are the first two digits. The product is 9894.

Volume of Prisms [#41-60]

A prism is a 3-dimensional solid with congruent bases on the top and bottom, separated by a distance we call the *height* or *altitude* of the prism. Prisms can be *right* prisms or *slant* prisms, as shown. The bases are allowed to be any 2-dimensional shape, but the top base must be congruent to the bottom base.



Right Prism Slant Prism

The formula for the volume V of a prism is $V = Bh$, where B is the area of the base and h is the height. Notice that this formula is true for any base shape. When the base is a common geometric shape, we have simplified formulas. For example, if the prism is a rectangular prism, the base is a rectangle. The area B of a rectangle is length times width, $\ell \times w$. Thus, the formula for the volume of a rectangular prism is $V = Bh = (\ell \times w) \times h = \ell wh$, a common formula that you probably already know.

Example:

Find the volume of a right triangular prism whose base has legs of 8 and 10 and whose height is 5.

Solution:

First, compute the area of the base. The area of a triangle is $\frac{1}{2}bh$, where b is the base and h is the height (of the triangle). Since we have a right triangle, the base and height are the same as the legs. This gives $B = \frac{1}{2}(8)(10) = 40$. To find the volume of the prism, multiply this area by the height of the prism: $40(5) = 200$.

Working with $x + y$ and xy [#61-80]

There are some interesting relationships that hold between two variables when we add them together, multiply them together, and when we have the sum of powers of the variables. Here are some of these formulas:

$$(x+y)^2 = (x^2+y^2) + 2xy \qquad \text{and} \qquad (x+y)^3 = (x^3+y^3) + 3(xy)(x+y)$$

$$x^2+y^2 = (x+y)^2 - 2xy \qquad \text{and} \qquad x^3+y^3 = (x+y)^3 - 3(xy)(x+y)$$

I will leave it to you to derive these formulas using a little algebra.

Example:

If $x + y = 10$ and $xy = 16$, find the value of $x^2 + y^2$.

Solution:

Using the first formula, we know that $x^2 + y^2 = (x+y)^2 - 2xy$. This gives $x^2 + y^2 = 10^2 - 2(16) = 100 - 32 = 68$.

Means [#61-80]

The math word *mean* means average, but in fact, there are different types of means. Here are the definitions and formulas for three means:

Means

Arithmetic Mean	**Geometric Mean**	**Harmonic Mean**
Add up all terms and divide by number of terms	Multiply all terms and take the nth root	Take the reciprocal of the arithmetic mean of the reciprocals of the terms
$\dfrac{x_1 + x_2 + x_3 + \ldots + x_n}{n}$	$\sqrt[n]{x_1 \cdot x_2 \cdot x_3 \cdot \ldots \cdot x_n}$	$\dfrac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \ldots + \frac{1}{x_n}}$

For the Harmonic Mean, there are short-cuts for when $n = 2$ and $n = 3$: For $n = 2$ and numbers a and b, the harmonic mean $H_2 = \dfrac{2ab}{a+b}$. For $n = 3$ and numbers a, b, and c, the harmonic mean $H_3 = \dfrac{3abc}{ab+bc+ca}$.

Example:

Find the harmonic mean between 3 and 5.

Solution:

Use the short-cut formula for H_2: $H_2 = \dfrac{2(3)(5)}{3+5} = \dfrac{30}{8} = \dfrac{15}{4}$.

Example:

Find the geometric mean of 18, 6, and 16.

Solution:

The geometric mean of the three numbers a, b, and c is $\sqrt[3]{abc}$. Since you need to take the cube root, look for sets of three factors that can come out of the cube root. $G_3 = \sqrt[3]{(18)(6)(16)} = \sqrt[3]{(3\cdot 3\cdot 2)(3\cdot 2)(2\cdot 2\cdot 2\cdot 2)}$. There are three 3's, so the answer has one 3. There are six 2's, so the answer must have two 2's in the product: $3\cdot 2\cdot 2 = 12$.

Practice Questions – The following practice questions cover the above examples and should be used to guide your inquiries into the new types of questions to be asked on the Number Sense tests.

1. $98 \times 105 =$ _____
2. $103 \times 96 =$ _____
3. $104 \times 97 =$ _____
4. $95 \times 108 =$ _____
5. Find the volume of a prism whose base area is 60 cm^2 and height is 9 cm. _____ cm^3
6. The volume of a prism is 240 in^3. Its height is 8 in. What is the area of its base? _____ in^2
7. A prism has a rhombus for a base. The diagonals of the rhombus measure 6" and 8". The height of the prism is 12". Find the volume of the prism. _____ in^3
8. A prism with a trapezoidal base has a height of 5 feet. The trapezoid's bases are 11 feet and 3 feet and its height is 6 feet. What is the volume of the prism? _____ ft^3
9. If $x + y = 9$ and $xy = 12$, what is $x^2 + y^2$? _____
10. If $x^2 + y^2 = 34$ and $x + y = 8$, then $xy =$ _____
11. If $x + y = 5$ and $xy = 4$, what is $x^3 + y^3$? _____
12. If $xy = 20$ and $x + y = 10$, then $x^3 + y^3 =$ _____
13. Find the geometric mean between 12 and 3. _____
14. Find the geometric mean of 1, 1, 1, and 81. _____
15. Find the harmonic mean between 10 and 12. _____
16. What is the harmonic mean of 2, 3, and 6? _____

Answers:	1. 10290	2. 9888	3. 10088	4. 10260	5. 540	6. 30	7. 288
	8. 210	9. 57	10. 15	11. 65	12. 400	13. 6	14. 3	15. 120/11	16. 3
